AP® Exam Practice Questions for Chapter 8

1. $u = 2 + \cos x \Rightarrow du = -\sin x\,dx$

$$\int (\sin x)e^{2+\cos x}\,dx = -\int e^u\,du = -e^u + C = -e^{2+\cos x} + C$$

So, the answer is A.

2. $$\frac{9x}{(3x-2)(x+4)} = \frac{A}{3x-2} + \frac{B}{x+4}$$

$$9x = A(x+4) + B(3x-2)$$

When $x = -4$, $-14B = -36 \Rightarrow B = \frac{18}{7}$.

When $x = \frac{2}{3}$, $\frac{14}{3}A = 6 \Rightarrow A = \frac{9}{7}$.

$$\int \frac{9x}{(3x-2)(x+4)}\,dx = \frac{9}{7}\int \frac{1}{3x-2}\,dx + \frac{18}{7}\int \frac{1}{x+4}\,dx$$

$$= \frac{9}{7}\cdot\frac{1}{3}\ln|3x-2| + \frac{18}{7}\ln|x+4| + C$$

$$= \frac{3}{7}\ln|3x-2| + \frac{18}{7}\ln|x+4| + C$$

So, the answer is A.

3. $u = f(x) \Rightarrow du = f'(x)\,dx$

$dv = g'(x)\,dx \Rightarrow v = g(x)$

$$\int_1^2 f(x)g'(x)\,dx = \left[f(x)g(x)\right]_1^2 - \int_1^2 f'(x)g(x)\,dx$$

$$= \left[f(2)g(2) - f(1)g(1)\right] - 3$$

$$= (2)(3) - (3)(-1) - 3$$

$$= 6$$

So, the answer is D.

4. $$\frac{1}{x^2 - 9x + 20} = \frac{1}{(x-5)(x-4)} = \frac{A}{x-5} + \frac{B}{x-4}$$

$$1 = A(x-4) + B(x-5)$$

When $x = 4$, $B = -1$.

When $x = 5$, $A = 1$.

$$\int_0^3 \frac{1}{x^2 - 9x + 20}\,dx = \int_0^3 \frac{1}{x-5}\,dx - \int_0^3 \frac{1}{x-4}\,dx$$

$$= \left[\ln|x-5| - \ln|x-4|\right]_0^3$$

$$= \left[\ln\left|\frac{x-5}{x-4}\right|\right]_0^3$$

$$= \ln 2 - \ln\frac{5}{4}$$

$$= \ln\frac{2}{\frac{5}{4}}$$

$$= \ln\frac{8}{5}$$

So, the answer is A.

5. Use the Trapezoidal Rule to find an approximation of $\int_0^4 g(x)\,dx$.

$$\int_0^4 g(x)\,dx \approx \frac{4-0}{2(8)}\left[0 + 2(3) + 2(7) + 2(12) + 2(18) + 2(25) + 2(33) + 2(42) + 52\right] = \frac{1}{4}(332) = 83$$

So, the answer is B.

6. $\lim_{x \to 2} \dfrac{\int_2^x e^{t/2}\,dt}{x^3 - 8} = \lim_{x \to 2} \dfrac{e^{x/2}}{3x^2}$

$= \dfrac{e^1}{3(2)^2}$

$= \dfrac{e}{12}$

So, the answer is C.

7. $\int_1^\infty \dfrac{dx}{x^{5/2}} = \lim_{b \to \infty} \int_1^b x^{-5/2}\,dx$

$= \lim_{b \to \infty} \left[-\dfrac{2}{3}x^{-3/2}\right]_1^b$

$= -\dfrac{2}{3} \lim_{b \to \infty} \left(\dfrac{1}{b^{3/2}} - 1\right)$

$= -\dfrac{2}{3}(0 - 1)$

$= \dfrac{2}{3}$

So, the answer is C.

8. $\int_2^8 f(x)\,dx = \int_2^3 f(x)\,dx + \int_3^5 f(x)\,dx + \int_5^8 f(x)\,dx$

$\approx \dfrac{3-2}{2}\left[f(2) + f(3)\right] + \dfrac{5-3}{2}\left[f(3) + f(5)\right] + \dfrac{8-5}{2}\left[f(5) + f(8)\right]$

$= \dfrac{1}{2}(8 + 22) + 1(22 + 72) + \dfrac{3}{2}(72 + 142)$

$= 430$

So, the answer is C.

9. $\int_0^\infty xe^{-x/3}\,dx = \lim_{b \to \infty} \int_0^b xe^{-x/3}\,dx$

$u = x \quad \Rightarrow \quad du = dx$

$dv = e^{-x/3}\,dx \quad \Rightarrow \quad v = -3e^{x/3}$

$= \lim_{b \to \infty} \left(\left[-3xe^{-x/3}\right]_0^b - \int_0^b -3e^{-x/3}\,dx\right)$

$= \lim_{b \to \infty} \left(\left[-3xe^{-x/3}\right]_0^b + 3\left[-3e^{-x/3}\right]_0^b\right)$

$= \lim_{b \to \infty} \left[\left(-3be^{-b/3} - 0\right) - 9\left(e^{-b/3} - e^0\right)\right]$

$= 0 - 9(0 - 1)$

$= 9$

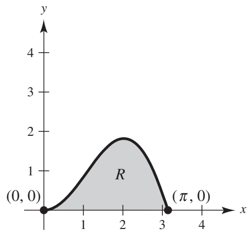
So, the answer is C.

10. $V = 2\pi \int_0^1 xe^{-x^3}\,dx$

≈ 2.198

So, the answer is B.

11. (a)



$A = \int_0^{\pi} x \sin x \, dx$

$u = x \Rightarrow du = dx$

$dv = \sin x \, dx \Rightarrow v = -\cos x$

$A = \left[-x \cos x\right]_0^{\pi} - \int_0^{\pi} -\cos x \, dx$

$= \left[-x \cos x + \sin x\right]_0^{\pi}$

$= \left[-\pi(-1) + 0\right] - (0 - 0)$

$= \pi$

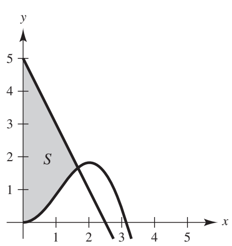
So, the area is π.

3 pts: { 2 pts: finds antiderivative (using parts); 1 pt: answer }

Notes: Be sure to justify the antiderivative by explicitly showing integration by parts—as per the given directions. (Merely writing down the definite integral and numerically approximating the area on your calculator will not receive full credit here because the directions say to use integration by parts.)

If presenting the answer as a decimal approximation, be sure to round the answer to at least three decimal places to receive credit on the exam.

(b)



Find the intersection of the graphs.

$x \sin x = -2x + 5$

$x \approx 1.6693678$

$A = \int_0^{1.6693678} \left[(-2x + 5) - (x \sin x)\right] dx$

$= \int_0^{1.6693678} (-2x + 5)\, dx - \int_0^{1.6693678} x \sin x \, dx$

$u = x \Rightarrow du = dx$

$dv = \sin x \, dx \Rightarrow v = -\cos x$

$= \left[-x^2 + 5x\right]_0^{1.6693678} - \left[-x \cos x\right]_0^{1.6693678} + \int_0^{1.6693678} -\cos x \, dx$

$= \left[-x^2 + 5x + x \cos x - \sin x\right]_0^{1.6693678}$

≈ 4.401

3 pts: { 2 pts: writes integral; 1 pt: answer (no work needed) }

Notes: Use your calculator to find the point of intersection of the two graphs (no work needed for this).

In this intermediate step, round the coordinates of the intersection point to more than three decimal places to use in upcoming intergrals.

Because the directions do not specify otherwise, it is sufficient here to simply write down the definite integral and then numerically approximate the area on your calculator.

Be sure to round the answer to at least three decimal places to receive credit on the exam.

(c) $\int_0^{1.6693678} (-2x + 5 - x \sin x)\, dx$

$= 2 \int_0^{k} (-2x + 5 - x \sin x)\, dx$

3 pts: { 1 pt: integrand; 2 pts: limits of integration within correct equation }

12. (a)

$$x(t) = 2 + \int_0^2 v(t)\,dt$$

$$= 2 + \int_0^2 \left(te^{-t/3} - \frac{1}{2}\right) dt$$

$$= 2 + \int_0^2 te^{-t/3}\,dt - \frac{1}{2}\int_0^2 dt$$

$$u = t \quad \Rightarrow \quad du = dt$$

$$dv = e^{-t/3}\,dt \quad \Rightarrow \quad v = -3e^{-t/3}$$

$$= 2 + \left[-3te^{-t/3}\right]_0^2 + 3\int_0^2 e^{-t/3}\,dt - \frac{1}{2}[t]_0^2$$

$$= 2 + \left[-3te^{-t/3} - 9e^{-t/3}\right]_0^2 - 1$$

$$= 1 + \left[\left(-3(2)e^{-2/3} - 9e^{-2/3}\right) - (0 - 9)\right]$$

$$= 10 - \frac{15}{e^{2/3}}$$

$$\approx 2.299$$

5 pts:
- 1 pt: uses initial condition
- 3 pts: finds antiderivative (using parts)
- 1 pt: answer

Notes: Be sure to justify the antiderivative by explicitly showing integration by parts—as per the given directions. (Merely writing down the definite integral and numerically approximating the area on your calculator will not receive full credit here because the directions say to use integration by parts.)

Be sure to round your answer to at least three decimal places to receive credit on the exam.

(b) $\int_0^2 \left|te^{-t/3} - \frac{1}{2}\right| dt$

2 pts: integral

(c)

$$a(t) = v'(t)$$

$$= e^{-t/3} + t\left(-\frac{1}{3}e^{-t/3}\right) - 0$$

$$= e^{-t/3}\left(1 - \frac{t}{3}\right)$$

Because $a(3) = e^{-3/3}\left[1 - \frac{3}{3}\right] = 0$, the particle's speed is not changing at $t = 3$.

2 pts:
- 1 pt: computes $a(3) = v'(3) = 0$ (no work needed)
- 1 pt: answer ("neither") with reason

13. (a) $f'(x) = \dfrac{1}{x(\ln x)^3}$

Because $f'(e) = \dfrac{1}{e(\ln e)^3} = \dfrac{1}{e}$, the equation of the tangent line at $(e, 4)$ is

$$y - 4 = \frac{1}{e}(x - e)$$

$$y = \frac{1}{e}x + 3.$$

2 pts: { 1 pt: finds the slope of the tangent line $[f'(e)]$; 1 pt: writes an equation of the tangent line }

(b) $f'(x) = \dfrac{1}{x(\ln x)^3} = \dfrac{1}{x}(\ln x)^{-3}$

$$f''(x) = \frac{1}{x}\left[-3(\ln x)^{-4} \cdot \frac{1}{x}\right] + \left(-\frac{1}{x^2}\right)(\ln x)^{-3}$$

$$= -\frac{3}{x^2(\ln x)^4} - \frac{1}{x^2(\ln x)^3}$$

$$f''(e) = -\frac{3}{e^2(\ln e)^4} - \frac{1}{e^2(\ln e)^3} \approx -0.541 < 0$$

So, the graph of f is concave downward on $(1, 5)$ because $f''(x) < 0$ on $(1, 5)$.

4 pts: { 2 pts: finds $f''(x)$; 2 pts: answer (concave down) with reason [examines sign of $f''(x)$ on this interval] }

Notes: After finding $f''(x)$, you could simply state that $f''(x)$ is negative for all x-values on $(1, 5)$; a sign chart may not be necessary.

If using a sign chart as part of the justification, the functions $f''(x)$ and f must be explicitly labeled in your chart. Unlabeled sign charts may not receive credit on the exam.

A sign chart alone is generally *not* sufficient for the explanation. To receive full credit on the exam, explain the information contained in the sign chart. For example, state "f is concave down on $(1, 5)$ because $f''(x)$ is negative on this interval."

(c) $f(x) = \displaystyle\int f'(x)\,dx = \int \frac{1}{x(\ln x)^3}\,dx$

$$u = \ln x \Rightarrow du = \frac{1}{x}\,dx$$

$$\int \frac{1}{x(\ln x)^3}\,dx = \int u^{-3}\,du$$

$$= -\frac{1}{2}u^{-2} + C$$

$$= -\frac{1}{2(\ln x)^2} + C$$

Use $(e, 4)$ to find C.

$$f(e) = -\frac{1}{2(\ln e)^2} + C = 4$$

$$-\frac{1}{2} + C = 4$$

$$C = \frac{9}{2}$$

So, $f(x) = -\dfrac{1}{2(\ln x)^2} + \dfrac{9}{2}$.

3 pts: { 2 pts: antiderivative (shows substitution); 1 pt: uses $f(e) = 4$ to find constant of integration and particular solution }

14. (a) $f(0) = \dfrac{3 + 4(0)}{1 + 0^2} = 3$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{3 + 4x}{1 + x^2} = 3$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \left(\tan^2 x + 3\right) = 3$$

So, $\lim_{x\to 0} f(x) = 3.$

Because $f(0) = \lim_{x\to 0} f(x)$, $f(x)$ is continuous at $x = 0$.

2 pts:
- 1 pt: considers each one-sided limit to find $\lim_{x\to 0} f(x)$
- 1 pt: shows $f(0) = \lim_{x\to 0} f(x)$ to reach conclusion

(b) $$f(x) = \begin{cases} \dfrac{3 + 4x}{1 + x^2}, & \text{for } x \le 0 \\ \tan^2 x + 3, & \text{for } x > 0 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{\left(1 + x^2\right)(4) - (3 + 4x)(2x)}{\left(1 + x^2\right)^2}, & \text{for } x \le 0 \\ 2 \tan x \sec^2 x, & \text{for } x > 0 \end{cases}$$

$$= \begin{cases} \dfrac{4 - 6x - 4x^2}{\left(1 + x^2\right)^2}, & \text{for } x \le 0 \\ 2 \tan x \sec^2 x, & \text{for } x > 0 \end{cases}$$

2 pts: antiderivatives for each piece
Note: You do not need to simplify these derivatives.

(c) $$\text{Average value} = \frac{1}{1 - (-2)}\left[\int_{-2}^{0} \frac{3 + 4x}{1 + x^2}\,dx + \int_0^1 \left(\tan^2 x + 3\right) dx\right]$$

$$= \frac{1}{3}\left[\int_{-2}^{0} \frac{3}{1 + x^2}\,dx + \int_{-2}^{0} \frac{4x}{1 + x^2}\,dx + \int_0^1 \left(\sec^2 x - 1 + 3\right) dx\right]$$

$$= \int_{-2}^{0} \frac{1}{1 + x^2}\,dx + \frac{2}{3}\int_{-2}^{0} \frac{2x}{1 + x^2}\,dx + \frac{1}{3}\int_0^1 \left(\sec^2 x + 2\right) dx$$

$$= \left[\arctan x\right]_{-2}^{0} + \frac{2}{3}\left[\ln\left(1 + x^2\right)\right]_{-2}^{0} + \frac{1}{3}\left[\tan x + 2x\right]_0^1$$

$$= \arctan 0 - \arctan(-2) + \frac{2}{3}(\ln 1 - \ln 5) + \frac{1}{3}(\tan 1 + 2 - \tan 0 - 0)$$

$$= -\arctan(-2) - \frac{2}{3}\ln 5 + \frac{1}{3}\tan 1 + \frac{2}{3}$$

5 pts:
- 2 pts: sets up both definite integrals as a sum
- 3 pts: antiderivatives, answer

15. (a) $\frac{1}{2R(5-R)} = \frac{1}{2}\left[\frac{A}{R} + \frac{B}{5-R}\right]$

$A(5-R) + BR = 1 \Rightarrow A = \frac{1}{5}$ and $B = \frac{1}{5}$

$$\int dT = \frac{1}{2}\int \frac{1}{R(5-R)}\,dR$$

$$T = \frac{1}{2}\int\left(\frac{1}{5R} + \frac{1}{5(5-R)}\right)dR$$

$$T = \frac{1}{10}\left(\ln|R| - \ln|5-R|\right) + C$$

$$T = \frac{1}{10}\ln\left|\frac{R}{5-R}\right| + C$$

Use $T = 0$ and $R = 3$ to find C.

$$0 = \frac{1}{10}\ln\left|\frac{3}{5-3}\right| + C$$

$$0 = \frac{1}{10}\ln\frac{3}{2} + C$$

$$C = -\frac{1}{10}\ln\frac{3}{2}$$

So, $T = \frac{1}{10}\ln\left|\frac{R}{5-R}\right| - \frac{1}{10}\ln\frac{3}{2}.$

5 pts:
- 3 pts: antiderivative (shows partial fraction decomposition)
- 1 pt: uses initial value
- 1 pt: answer

(b) $\frac{dT}{dR} = \frac{1}{2R(5-R)} = \frac{1}{10R - 2R^2}$

$$\frac{d^2T}{dR^2} = \frac{(10R - 2R^2)(0) - 1(10 - 4R)}{(10R - 2R^2)^2}$$

$$= \frac{4R - 10}{(10R - 2R^2)^2}$$

$$0 = \frac{4R - 10}{(10R - 2R^2)^2}$$

$$R = \frac{5}{2}$$

For $R = 2$, $\frac{d^2T}{dR^2} = \frac{4(2) - 10}{\left[10(2) - 2(2^2)\right]^2} = -\frac{1}{72}$

For $R = 3$, $\frac{d^2T}{dR^2} = \frac{4(3) - 10}{\left[10(3) - 2(3^2)\right]^2} = \frac{1}{72}$

Because $\frac{d^2T}{dR^2}$ changes sign at $R = \frac{5}{2}$, the graph of T has a point of inflection at $R = \frac{5}{2}$.

4 pts:
- 1 pt: finds $\frac{d^2T}{dR^2}$
- 1 pt: finds where $\frac{d^2T}{dR^2} = 0$
- 2 pts: answer with reason (examines sign of $\frac{d^2T}{dR^2}$ on both sides of $R = \frac{5}{2}$ to confirm T changes concavity)

Notes: In such an explanation, reason using $\frac{d^2T}{dR^2}$ (explain using a derivative). Merely reasoning with T (appealing to where T changes concavity) may not receive credit on the exam.

If using a sign chart as part of the justification, the functions $\frac{d^2T}{dR^2}$ and T must be explicitly labeled in your chart. Unlabeled sign charts may not receive credit on the exam. A sign chart alone is generally *not* sufficient for the explanation. To receive full credit on the exam, explain the information contained in the sign chart. For example, state "T has an inflection point at $R = \frac{5}{2}$ because $\frac{d^2T}{dR^2}$ changes sign at $R = \frac{5}{2}$."

16. (a) Because $\lim_{x\to\infty} \frac{x}{e^x} = \frac{\infty}{\infty}$, apply L'Hôpital's Rule.

$$\lim_{x\to+\infty} g(x) = \lim_{x\to+\infty} xe^{-x} = \lim_{x\to+\infty} \frac{x}{e^x} = \lim_{x\to+\infty} \frac{1}{e^x} = 0$$

Because $\lim_{x\to-\infty} \frac{x}{e^x} = \frac{-\infty}{0}$, L'Hôpital's Rule does not apply.

$$\lim_{x\to-\infty} g(x) = \lim_{x\to-\infty} xe^{-x} = (-\infty)(\infty) = -\infty$$

So, the graph of $g(x)$ has a horizontal asymptote at $y = 0$.

3 pts:
- 1 pt: considers both $\lim_{x\to\infty} g(x)$ and $\lim_{x\to-\infty} g(x)$
- 2 pts: applies L'Hôpital's Rule to evaluate the first limit $(x \to +\infty)$ but not the second limit $(x \to -\infty)$, answer

Note: Be sure to establish an indeterminant form, $\frac{0}{0}$ or $\frac{\infty}{\infty}$, to justify the use of L'Hôpital's Rule.

(b) $g(x) = xe^{-x}$

$$g'(x) = x(-e^{-x}) + e^{-x}(1)$$
$$= e^{-x}(-x+1)$$
$$= \frac{1-x}{e^x}$$
$$0 = \frac{1-x}{e^x}$$
$$x = 1$$

So, $g(1) = (1)e^{-(1)} = \frac{1}{e}$.

For $x = 0, g'(x) = \frac{1-0}{e^0} = 1$

For $x = 2, g'(x) = \frac{1-2}{e^2} = -\frac{1}{e^2}$

Because $g'(x)$ changes from positive to negative at $x = 1$, the graph has a maximum at $\left(1, \frac{1}{e}\right)$.

3 pts:
- 1 pt: finds where $g'(x) = 0$
- 2 pts: answer [finds $g(1)$] with reason [examines sign of $g'(x)$ on both sides of $x = 1$ to establish a relative maximum at the only critical value, $x = 1$]

Notes: In such an explanation, reason using $g'(x)$ (explain using a derivative). Merely reasoning with g (appealing to where g changes from increasing to decreasing) may not receive credit on the exam.

If using a sign chart as part of the justification, the function $g'(x)$ and g must be explicitly labeled in your chart. Unlabeled sign charts may not receive credit on the exam.

A sign chart alone is generally *not* sufficient for the explanation. To receive full credit on the exam, explain the information contained in the sign chart. For example, state "g has a relative maximum at $x = 1$ because $g'(x)$ changes from positive to negative at $x = 1$."

(c) $A = \int_1^\infty xe^{-x}\,dx = \lim_{b\to\infty} \int_1^b xe^{-x}\,dx$

$u = x \Rightarrow du = dx$

$dv = e^{-x}\,dx \Rightarrow v = -e^{-x}$

$$A = \lim_{b\to\infty}\left(\left[-xe^{-x}\right]_1^b - \int_1^b -e^{-x}\,dx\right)$$
$$= \lim_{b\to\infty}\left(\left[-xe^{-x} - e^{-x}\right]_1^b\right)$$
$$= \lim_{b\to\infty}\left[\left(-be^{-b} - e^{-b}\right) - \left[-(1)e^{-1} - e^{-1}\right]\right]$$
$$= \lim_{b\to\infty}\left(-\frac{b}{e^b} - \frac{1}{e^b} + \frac{1}{e} + \frac{1}{e}\right)$$
$$= 0 - 0 + \frac{1}{e} + \frac{1}{e}$$
$$= \frac{2}{e}$$

3 pts:
- 1 pt: sets up and rewrites improper integral
- 1 pt: antiderivative (using parts)
- 1 pt: answer (evaluates limit using L'Hôpital's Rule)

Note: Be sure to establish an indeterminant form, $\frac{0}{0}$ or $\frac{\infty}{\infty}$, to justify the use of L'Hôpital's Rule.

17. (a) $\int_0^3 g'(x)\,dx = \int_0^2 g'(x)\,dx + \int_2^3 g'(x)\,dx$

$$= \lim_{b\to 2^-}\int_0^b (x-2)^{-2}\,dx + \lim_{b\to 2^+}\int_b^3 (x-2)^{-2}\,dx$$

$$= \lim_{b\to 2^-}\left[-(x-2)^{-1}\right]_0^b + \lim_{b\to 2^+}\left[-(x-2)^{-1}\right]_b^3$$

$$= \lim_{b\to 2^-}\left(-\frac{1}{b-2}-\frac{1}{2}\right) + \lim_{b\to 2^+}\left(-1+\frac{1}{b-2}\right)$$

$$= \infty + \infty$$

$$= \infty$$

So, $\int_0^3 g'(x)\,dx$ diverges.

5 pts:
- 2 pts: splits improper integral into two pieces (accounts for discontinuity at $x = 2$) with correct one-sided limits
- 1 pt: antiderivative
- 2 pts: shows each one-sided limit goes to infinity, answer

(b) Because there is a discontinuity in $[0, 3]$ at $x = 2$, the Fundamental Theorem of Calculus does not apply.

1 pt: reason [identifies $g'(x)$ has a discontinuity in $[0, 3]$ at $x = 2$]

(c) $\int_4^\infty g'(x)\,dx = \lim_{b\to\infty}\int_4^b g'(x)\,dx$

$$= \lim_{b\to\infty}\int_4^b (x-2)^{-2}\,dx$$

$$= \lim_{b\to\infty}\left[-(x-2)^{-1}\right]_4^b$$

$$= \lim_{b\to\infty}\left[-\frac{1}{b-2}+\frac{1}{2}\right]$$

$$= 0 + \frac{1}{2} = \frac{1}{2}$$

3 pts:
- 1 pt: rewrites improper integral with limit
- 1 pt: antiderivative
- 1 pt: answer (evaluates limit)

18. (a) Because $\lim_{x \to 0} \dfrac{f(x) + 2}{\tan x} = \dfrac{0}{0}$, apply L'Hôpital's Rule.

$$\lim_{x \to 0} \frac{f'(x)}{\sec^2 x} = \frac{f'(0)}{1} = (-2)^2[4(0) + 1] = 4$$

2 pts: {1 pt: applies L'Hôpital's Rule; 1 pt: answer}

Note: Be sure to establish an indeterminant form, $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$, to justify the use of L'Hôpital's Rule.

(b) $f\left(\frac{1}{4}\right) \approx -2 + \frac{1}{4}(4) = -1$

$f\left(\frac{1}{2}\right) \approx -1 + \frac{1}{4}(2) = -\frac{1}{2}$

So, $f\left(\frac{1}{2}\right) \approx -\frac{1}{2}$.

2 pts: {1 pt: Euler's Method with two steps; 1 pt: answer $\left[\text{approximation of } f\left(\frac{1}{2}\right)\right]$}

Note: Be sure to write "$f\left(\frac{1}{2}\right) \approx$" rather than "$f\left(\frac{1}{2}\right) =$." Because this is an approximation, a point may be deducted if an equal sign is used. In general, equating two quantities that are not truly equal will result in a one point deduction on a free-response question.

(c) $\dfrac{dy}{dx} = y^2(4x + 1)$

$$\int \frac{1}{y^2}\, dy = \int (4x + 1)\, dx$$

$$-\frac{1}{y} = 2x^2 + x + C$$

Use $f(0) = -2$ to find C.

$$-\frac{1}{(-2)} = 2(0)^2 + 0 + C \Rightarrow C = \frac{1}{2}$$

Because $-\dfrac{1}{y} = 2x^2 + x + \dfrac{1}{2}$,

$$f(x) = -\left(2x^2 + x + \frac{1}{2}\right)^{-1}.$$

5 pts: {1 pt: separation of variables; 1 pt: antiderivatives; 1 pt: constant of integration; 1 pt: uses intitial condition; 1 pt: particular solution (solves for y)}

Notes: 2 points max if no constant of integration present

0 points if no separation of variables