AP® Exam Practice Questions for Chapter 3

1. $f(x) = 4x^3 + 6x^2 - 72x - 9$

$f'(x) = 12x^2 + 12x - 72 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, 2$

The critical numbers of $f(x)$ are $x = -3$ and $x = 2$.

So, the answer is B.

2. Evaluate each point.

A: $\frac{dy}{dx} < 0$, $\frac{d^2y}{dx^2} > 0$ B: $\frac{dy}{dx} > 0$, $\frac{d^2y}{dx^2} > 0$

C: $\frac{dy}{dx} > 0$, $\frac{d^2y}{dx^2} < 0$ D: $\frac{dy}{dx} < 0$, $\frac{d^2y}{dx^2} < 0$

So, the answer is C.

3. $s(t) = -t^3 + 3t^2 + 9t + 5$

$s'(t) = -3t^2 + 6t + 9$

$s''(t) = -6t + 6$

$s''(t) = 0$

$-6t + 6 = 0$

$6t = 6$

$t = 1$

So, $t = 1$ is a point of inflection of $s(t)$.

Use $s'(t)$ to find the velocity at $t = 1$.

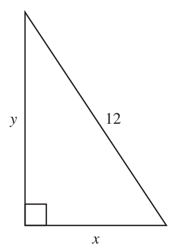
$s'(1) = -3(1)^2 + 6(1) + 9$

$= 12$

The maximum velocity is 12 feet per second.

So, the answer is B.

4.



$x^2 + y^2 = 12^2$

$y^2 = 144 - x^2$

$y = \pm\sqrt{144 - x^2}$ (Because $y > 0$, use $y = \sqrt{144 - x^2}$.)

Let A be the area to be maximized.

$A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{144 - x^2},\ 0 < x < 12$

$\frac{dA}{dx} = \frac{1}{2}x\left[\frac{1}{2}(144 - x^2)^{-1/2} \cdot (-2x)\right] + \frac{1}{2}\sqrt{144 - x^2} = \frac{\sqrt{144 - x^2}}{2} - \frac{x^2}{2\sqrt{144 - x^2}}$

$\frac{dA}{dx} = 0$

$\frac{\sqrt{144 - x^2}}{2} = \frac{x^2}{2\sqrt{144 - x^2}}$

$(144 - x^2) = x^2$

$2x^2 = 144$

$x = \pm\sqrt{72} = \pm 6\sqrt{2}$ (Because $x > 0$, use $x = 6\sqrt{2}$.)

$y = \sqrt{144 - (6\sqrt{2})^2} = \sqrt{72} = 6\sqrt{2}$

The maximum area is $A = \frac{1}{2}(6\sqrt{2})(6\sqrt{2}) = 36$ square units.

So, the answer is C.

5. Evaluate each statement.

A: The point $(4, 1)$ appears to be a relative minimum, but there may be another number c on $[2, 6]$ for which $g'(c) = 0$.

The statement may not be true.

B: The point $(6, 7)$ appears to be a relative maximum, but there may be another number c on $[2, 6]$ for which $g'(c) = 0$.

The statement may not be true.

C: Because g is continuous and differentiable on $[2, 6]$ and $g(2) = g(6)$, then there is at least one number c in $(2, 6)$ such that $g'(c) = 0$.

By Rolle's Theorem, this statement must be true.

D: The graph of g appears to be decreasing on $(2, 4)$, but there may be a point on $(2, 4)$ at which $g'(x) = 0$.

The statement may not be true.

So, the answer is C.

6. Evaluate each statement.

A: $f(10)$ may or may not be undefined based on the function f.

The statement may or may not be true.

B: $\lim_{x\to 10} f(x)$ may or may not exist based on the function f.

The statement may or may not be true.

C: Because $y = 10$ is a horizontal asymptote, $\lim_{x\to\infty} f(x) = 10$.

The statement must be true.

D: Even though $y = 10$ is a horizontal asymptote, there may be at least one value of x for which $f(x) = 10$.

The statement may or may not be true.

So, the answer is C.

7. Evaluate each statement.

I. Because $\lim_{x\to 4} f(x) = 2$ and $f(4) = 8$, f is not continuous at $x = 4$.

The statement is true.

II. $$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2 + 4x - 32}{x^2 - 2x - 8} = \lim_{x\to\infty} \frac{1 + \frac{4}{x} - \frac{32}{x^2}}{1 - \frac{2}{x} - \frac{8}{x^2}} = \frac{1}{1} = 1 \neq 4$$

The statement is false.

III. $$f(x) = \frac{x^2 + 4x - 32}{x^2 - 2x - 8} = \frac{(x + 8)(x - 4)}{(x - 4)(x + 2)} = \frac{x + 8}{x + 2},\ x \neq 4$$

f has a removable discontinuity at $x = 4$, not a vertical asymptote.

The statement is false.

Because I is the only statement that is true, the answer is A.

8. $y = f(c) + f'(c)(x - c)$

$y = f(3) + f'(3)(x - 3)$

$y = 8 + 22(x - 3)$

$y = 22x - 58$

$f(2.9) = 22(2.9) - 58 = 5.8$

So, the answer is B.

9. $f(x) = x^3 + 4x^2 + 3x - \cos x$

$f'(x) = 3x^2 + 8x + 3 + \sin x$

$f''(x) = 6x + 8 + \cos x$

$f''(x) = 6x + 8 + \cos x = 0$ when $x \approx -1.367$, which is the only point of inflection of the graph of f.

So, the answer is B.

10. $$\lim_{x\to\infty} \frac{x^3}{\sqrt{x^2 + 4}} = \lim_{x\to\infty} \frac{\frac{x^3}{x^3}}{\sqrt{\frac{x^2}{x^6} + \frac{4}{x^6}}} = \lim_{x\to\infty} \frac{1}{\sqrt{\frac{1}{x^4} + \frac{4}{x^6}}} = \frac{1}{0} = \infty$$

The answer is D.

11. (a) $f(x) = \dfrac{x^3}{2} - \sin x + 1$

$f'(x) = \dfrac{3}{2}x^2 - \cos x$

Using a graphing utility,

$f'(x) = \dfrac{3}{2}x^2 - \cos x = 0$ when $x \approx \pm 0.7108$.

So, the relative extrema of f occur at $x \approx \pm 0.7108$.

3 pts: { 1 pt: computes $f'(x)$; 2 pts: answers (finds critical numbers from calculator, no work needed) }

Notes: You would be expected to compute/work with $f'(x)$ here in justifying the critical points. Merely obtaining the critical points from your calculator would not receive full credit on the exam.

Round each answer to at least three decimal places to receive credit on the exam.

(b) $f\left(\dfrac{\pi}{2}\right) = \dfrac{1}{2}\left(\dfrac{\pi}{2}\right)^3 - \sin\left(\dfrac{\pi}{2}\right) + 1$

$= \dfrac{\pi^3}{16} - 1 + 1$

$= \dfrac{\pi^3}{16}$

$f'\left(\dfrac{\pi}{2}\right) = \dfrac{3}{2}\left(\dfrac{\pi}{2}\right)^2 - \cos\dfrac{\pi}{2}$

$= \dfrac{3\pi^2}{8}$

Tangent line: $y - \dfrac{\pi^3}{16} = \dfrac{3\pi^2}{8}\left(x - \dfrac{\pi}{2}\right)$

$y = \dfrac{3\pi^2}{8}x - \dfrac{3\pi^3}{16} + \dfrac{\pi^3}{16}$

$y = \dfrac{3\pi^2}{8}x - \dfrac{\pi^3}{8}$

4 pts: { 1 pt: finds $f\left(\dfrac{\pi}{2}\right)$; 2 pts: finds $f'\left(\dfrac{\pi}{2}\right)$; 1 pt: finds the equation of tangent line, solves for y }

(c) Using the tangent line approximation,

$f(1.5) \approx \dfrac{3\pi^2}{8}(1.5) - \dfrac{\pi^3}{8} \approx 1.676.$

The actual value of

$f(1.5) = \dfrac{(1.5)^3}{2} - \sin(1.5) + 1 \approx 1.690.$

So, the tangent line approximation is an underestimate of $f(1.5)$.

2 pts: { 1 pt: approximates $f(1.5)$; 1 pt: compares approximation with $f(1.5)$ and reaches conclusion }

Notes: An alternate explanation may be to identify that the tangent line at $x = \pi/2$ is below the graph of f. To see this, analyze the sign of $f''(\pi/2)$ to determine the concavity of f at $x = \pi/2$.

When using the tangent line to *approximate* $f(1.5)$, be sure to write "$f(1.5) \approx$" rather than "$f(1.5) =$." Because this is an approximation, a point may be deducted if an equal sign is used. In general, equating two quantities that are not truly equal will result in a one point deduction on a free-response question.

Be sure to round each answer to at least three decimal places to receive credit on the exam, and avoid premature rounding in intermediate steps.

Be sure your calculator is in *radian* mode.

12. (a) Because $f'(x) > 0$ when $x < 4$, f is increasing on the interval $(-\infty, 4)$.

2 pts: { 1 pt: answer (ignore inclusion of endpoint)
1 pt: justification [appeals to where $f'(x) > 0$] }

(b) Yes. Because f is continuous and $f'(x)$ changes from positive to negative at $x = 4$, f has a relative maximum at $x = 4$.

2 pts: answer with justification [appeals to $f'(x)$ changing from positive to negative at $x = 4$]

Note: In such an explanation, use $f'(x)$ in your justification (explain using a derivative). Merely reasoning with f (appealing to where f changes from increasing to decreasing) may not receive credit on the exam.

(c) Because f is continuous, $f(2) = 4$, $f''(x) < 0$ on $(-\infty, 4)$, and $f''(x) > 0$ on $(4, \infty)$, the point of inflection is at $x = 4$.

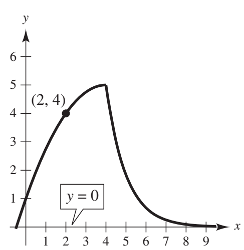
2 pts: answer with justification [appeals to $f''(x)$ changing sign at $x = 4$]

Note: Be sure to use $f''(x)$ in your justification (explain using a derivative) rather than reasoning with the concavity of f.

(d) No, $f(x)$ is not differentiable on $(3, 5)$.

1 pt: answer with justification

(e) Answers will vary. *Sample answer:*



2 pts: { 1 pt: graph reflects appropriate increasing/decreasing and concavity behavior
1 pt: appropriate behavior at $x = 4$ [relative max, $f'(x)$ undefined] }

Note: In the explanations throughout this question, be sure to explicitly identify each function by name. For example, referring to $f'(x)$ in part (a) as "it" or "the function" may not receive credit on the exam because there are three functions involved in the analysis of this question.

13. (a) Because $f'(x) > 0$ when $0 < x < 2$, f is increasing on the interval $(0, 2)$.

2 pts: answer with justification [appeals to $f'(x) > 0$]

Note: In your justification, be sure to explicitly identify each function by name. Referring to $f'(x)$ as "it," "the function," or "the graph" may not receive credit on the exam.

(b) Because $f''(-0.8) = 0$ and $f''(1.3) = 0$, the graph of f has points of inflection at $x = -0.8$ and $x = 1.3$. The graph of f' is decreasing when $-2 < x < -0.8$ and $x > 1.3$, so the graph of f is concave downward on $(-2, -0.8)$ and $(1.3, \infty)$.

4 pts: $\begin{cases} \text{2 pts: intervals} \\ \text{2 pts: justification [appeals to where } f'(x) \text{ is decreasing]} \end{cases}$

Note: In your justification, be sure to explicitly identify each function by name.

(c) By the Mean Value Theorem, there exists a number c in $(-0.5, 0)$ such that

$$\frac{f(-0.5) - f(0)}{-0.5 - 0} = f'(c).$$

Because $f'(c) < 0$ for $-0.5 < c < 0$,

$$\frac{f(-0.5) - f(0)}{-0.5 - 0} < 0.$$

3 pts: answer with justification

Note: In addition to reasoning with the Mean Value Theorem, an alternate explanation may involve justifying the sign of the numerator in the given difference quotient. Because $f'(x) < 0$ on $[-0.5, 0]$ from the given graph, f is decreasing on this interval. So, $f'(-0.5) > f(0)$, and the numerator of this difference quotient must be positive. With a positive numerator and negative denominator, the difference quotient itself must be negative.

14. (a) $f(x) = \dfrac{1 - 4x^2}{x} = \dfrac{1}{x} - 4x$

$f'(x) = -\dfrac{1}{x^2} - 4$

There are no points at which $f'(x) = 0$. Use a table to test the critical number $x = 0$, where f' does not exist.

Interval	$-\infty < x < 0$	$0 < x < \infty$
Test value	$x = -1$	$x = 1$
Sign of $f'(x)$	$f'(-1) = -\frac{1}{2} < 0$	$f'(1) = -5 < 0$
Graph of f	Decreasing	Decreasing

So, f is decreasing on $(-\infty, 0)$ and $(0, \infty)$ because $f'(x)$ is negative on these intervals.

5 pts:
- 1 pt: computes $f'(x)$
- 1 pt: finds the critical number [recognizes that there are no points at which $f'(x) = 0$, but $f'(x)$ does exist when $x = 0$]
- 1 pt: justification [examines sign of $f'(x)$ on either side of the critical number]
- 2 pts: answer with justification [appeals to where $f'(x) < 0$]

Notes: For the justification in this particular example, you could simply identify that $f'(x)$ is negative for all nonzero x-values. A sign chart may not be necessary.

If using a sign chart as part of the justification, the functions $f'(x)$ and f must be explicitly labeled in your chart. Unlabeled sign charts may not receive credit on the exam.

A sign chart alone is generally *not* sufficient for the explanation. To receive full credit on the exam, be sure to explain the information contained in the sign chart.

(b) f is concave downward when $f''(x) < 0$.

$f'(x) = -\dfrac{1}{x^2} - 4$

$f''(x) = \dfrac{2}{x^3}$

$f''(x) < 0$ when $x < 0$. So, $f(x)$ is concave downward on $(-\infty, 0)$.

3 pts:
- 2 pts: computes $f''(x)$ and examines where $f''(x) < 0$
- 1 pt: answer

(c) Because $f''(x) \neq 0$ and $f'(x)$ does not exist when $x = 0$, the graph of f does not have any points of inflection.

1 pt: answer with justification

Note: In these justifications, be sure to explicitly identify each function by name. Referring to $f'(x)$ or $f''(x)$ as "it," "the function," or "the graph" may not receive credit on the exam.

15. (a) Use $f'(-3) = 0$ and $f'(0) = -1$ to find an equation of $f'(x)$ on $[-3, 0]$.

$$m = \frac{-1 - 0}{0 - (-3)} = -\frac{1}{3}$$

$$f'(x) - 0 = -\frac{1}{3}[x - (-3)]$$

$$f'(x) = -\frac{1}{3}x - 1$$

So, $f'(-1) = -\frac{1}{3}(-1) - 1 = -\frac{2}{3}$.

Because $m = -\frac{1}{3}$ at $f'(-1)$, $f''(-1) = -\frac{1}{3}$.

2 pts:
- 1 pt: finds $f'(-1)$ with justification (finds/uses an *equation* of the given line segment)
- 1 pt: finds $f''(-1)$ with justification (finds/uses the *slope* of the given line segment)

(b) On the interval $(-5, 0)$, $f'(-3) = 0$.

So, f has a critical number at $x = -3$.

Because $f'(x) > 0$ on $(-5, -3)$ and $f'(x) < 0$ on $(-3, 0)$, f has a relative maximum at the point where $x = -3$.

2 pts: answer with justification [identifies that $f'(x)$ changes from positive to negative at $x = -3$]

(c) The points of inflection occur at $x = -4$, $x = 0$, and $x = 1$ because $f''(-4) = 0$, $f''(0)$ and $f''(1)$ are undefined, and $f'(x)$ changes from either increasing to decreasing or decreasing to increasing at these x-values [see part (c)].

3 pts: answers with justification [identifies where $f''(x) = 0$ or where $f''(x)$ is undefined *and* that $f'(x)$ changes from increasing to decreasing or from decreasing to increasing at these x-values]

(d)

$$g(x) = f(x) + \sin^2 x$$

$$g'(x) = f'(x) + 2 \sin x \cos x$$

$$g'\left(-\frac{\pi}{4}\right) = f'\left(-\frac{\pi}{4}\right) + 2 \sin\left(-\frac{\pi}{4}\right)\cos\left(-\frac{\pi}{4}\right)$$

$$= f'\left(-\frac{\pi}{4}\right) + 2\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = f'\left(-\frac{\pi}{4}\right) - 1$$

From the graph, $f'\left(-\frac{\pi}{4}\right)$ is negative. So, $g'\left(-\frac{\pi}{4}\right)$ is negative, which means that g is decreasing at $x = -\frac{\pi}{4}$.

2 pts:
- 1 pt: computes $g'\left(-\frac{\pi}{4}\right)$
- 1 pt: answer with justification [identifies that $g'\left(-\frac{\pi}{4}\right)$ is negative]

Note: In these justifications, be sure to explicitly identify each function by name. For example, referring to $f'(x)$ or $f''(x)$ as "it," "the function," or "the graph" may not receive credit on the exam.