

AP[®] Exam Practice Questions for Chapter 2

1. Evaluate each statement.

A: “ f is continuous at $x = c$ ” is a true statement by Theorem 2.1.

B: “ $\lim_{x \rightarrow c} f(x)$ exists” is a true statement by the definition of a limit.

C: “ $f'(c)$ is defined” is a true statement by the definition of a derivative.

D: “ $f''(c)$ is defined” could be a false statement because f' may not be differentiable at $x = c$.

So, the answer is D.

2. Evaluate each graph.

A: The graph appears to be $f(x) = -2x$, so

$$f'(x) = -2.$$

B: The graph appears to be $f(x) = x^3$, so

$$f'(x) = 3x^2.$$

C: The graph appears to be $f(x) = x^2$, so

$$f'(x) = 2x.$$

D: The graph appears to be $f(x) = x$, so $f'(x) = 1$.

Because $f'(x) = -2$ is negative for all values of x , the answer is A.

$$\begin{aligned} 3. \quad y &= \frac{6x^4 - 3x^5 + 5x^3}{x^3} \\ &= 6x - 3x^2 + 5 \\ \frac{dy}{dx} &= 6 - 6x \\ \frac{d^2y}{dx^2} &= -6 \end{aligned}$$

So, the answer is D.

$$4. \quad f(x) = \frac{5}{2}\sqrt{x} \Rightarrow f'(x) = \frac{5}{4\sqrt{x}}$$

$$\text{Because } f'(3) = \frac{5}{4\sqrt{3}},$$

$$f'(c) = 2\left(\frac{5}{4\sqrt{3}}\right) = \frac{5}{2\sqrt{3}} = \frac{5\sqrt{3}}{6}.$$

$$\frac{5}{4\sqrt{c}} = \frac{5\sqrt{3}}{6}$$

$$30 = 20\sqrt{3c}$$

$$\frac{3}{2} = \sqrt{3c}$$

$$\frac{9}{4} = 3c$$

$$\frac{3}{4} = c$$

So, the answer is C.

5. The function $h(x) = |2x - 5|$ is continuous at $x = \frac{5}{2}$.

$$\text{Because } \lim_{x \rightarrow (5/2)^-} \frac{|2x - 5|}{x - (5/2)} \neq \lim_{x \rightarrow (5/2)^+} \frac{|2x - 5|}{x - (5/2)},$$

the function is not differentiable at $x = 5/2$.

So, the answer is A.

$$6. \quad y = \sqrt[4]{8x + 3} = (8x + 3)^{1/4}$$

$$y' = \frac{1}{4}(8x + 3)^{-3/4}(8) = \frac{2}{(8x + 3)^{3/4}}$$

So, the answer is A.

$$7. \quad f(x) = \sin x - \cos x$$

$$f'(x) = \cos x + \sin x = \sin x + \cos x$$

$$f''(x) = \cos x - \sin x = -\sin x + \cos x$$

$$f'''(x) = -\cos x - \sin x = -\sin x - \cos x$$

$$f^{(4)}(x) = -\cos x + \sin x = \sin x - \cos x = f(x)$$

Because $n = 4$, the answer is B.

$$8. \quad \frac{s(3.2) - s(2.7)}{3.2 - 2.7} = \frac{10.6 - 7.8}{3.2 - 2.7} = \frac{2.8}{0.5} = 5.6 \text{ m/sec}$$

So, the answer is C.

$$9. \quad 2y^3 - 3xy + x^2 = 4$$

$$6y^2 \frac{dy}{dx} - 3x \frac{dy}{dx} - 3y + 2x = 0$$

$$(-3x + 6y^2) \frac{dy}{dx} = -2x + 3y$$

$$\frac{dy}{dx} = \frac{-(2x - 3y)}{(-3x + 6y^2)}$$

$$\frac{dy}{dx} = \frac{2x - 3y}{3x - 6y^2}$$

So, the answer is B.

10. $V = \pi r^2 h$

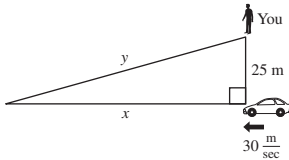
$$\frac{dV}{dt} = \pi \left[r^2 \left(\frac{dh}{dt} \right) + 2r \left(\frac{dr}{dt} \right) h \right] = \pi \left(r^2 \cdot \frac{dh}{dt} + 2rh \frac{dr}{dt} \right)$$

When $\frac{dr}{dt} = \frac{1}{3}$, $\frac{dh}{dt} = \frac{1}{2}$, $h = 9$, and $r = 4$,

$$\frac{dV}{dt} = \pi \left[(4)^2 \left(\frac{1}{2} \right) + 2(4)(9) \left(\frac{1}{3} \right) \right] = 32\pi \text{ cubic centimeters per second.}$$

So, the answer is D.

11.



$$x^2 + 25^2 = y^2$$

$$2x \frac{dx}{dt} + 0 = 2y \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt}$$

After 3 seconds, $x = 30(3) = 90$ meters and $y = \sqrt{90^2 + 25^2} = \sqrt{8725} = 5\sqrt{349}$.

Because $\frac{dx}{dt} = 30$ meters per second, $\frac{dy}{dt} = \frac{90}{5\sqrt{349}}(30) \approx 28.906$ meters per second.

So, the answer is B.

12. $s(t) = -t^3 + 2t^2 + \frac{3}{2}$

$$s'(t) = -3t^2 + 4t$$

$$\text{Average velocity} = \frac{s(4) - s(0)}{4 - 0} = \frac{[-(4)^3 + 2(4)^2 + \frac{3}{2}] - [-(0)^3 + 2(0)^2 + \frac{3}{2}]}{4} = \frac{-32}{4} = -8$$

$$s'(t) = -8$$

$$-3t^2 + 4t = -8$$

$$0 = 3t^2 - 4t - 8$$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$$

$$t = \frac{4 \pm \sqrt{112}}{6}$$

$$t \approx 2.431 \text{ seconds } (t \approx -1.097 \text{ is not in the domain.})$$

So, the answer is D.

13. (a) $v(t) = 2 + 3.5 \cos 0.7t$
 $v'(t) = 0 - 3.5(\sin 0.7t)(0.7)$
 $= -2.45 \sin 0.7t$
 which represents the acceleration of the particle.

6 pts: $\begin{cases} 4 \text{ pts: computes } v'(t) \\ 2 \text{ pts: interprets } v'(t) \end{cases}$

- (b) $a(5) = v'(5)$
 $= -2.45 \sin(0.7 \cdot 5)$
 ≈ 0.859

3 pts: answer

At $t = 5$, the acceleration of the particle is about 0.859.

14. (a)
$$\lim_{h \rightarrow 0} \frac{\sqrt{16+h} - 4}{h} \left(\frac{\sqrt{16+h} + 4}{\sqrt{16+h} + 4} \right) = \lim_{h \rightarrow 0} \frac{(16+h) - 16}{h(\sqrt{16+h} + 4)}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{16+h} + 4)}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{16+h} + 4}$$

$$= \frac{1}{\sqrt{16+4}} = \frac{1}{8}$$

5 pts: $\begin{cases} 3 \text{ pts: scales numerator and denominator by} \\ \text{strategic factor (conjugate of numerator)} \\ 2 \text{ pts: answer with analysis (simplifies,} \\ \text{cancels out factor of } h, \text{ evaluates limit)} \end{cases}$

(b)
$$\lim_{h \rightarrow 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} = \lim_{h \rightarrow 0} \frac{5 - 5 - h}{5(5+h)} \cdot \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-1}{5(5+h)}$$

$$= -\frac{1}{5(5+0)}$$

$$= -\frac{1}{25}$$

4 pts: $\begin{cases} 2 \text{ pts: manufacturers a common denominator} \\ 2 \text{ pts: answer with analysis (simplifies,} \\ \text{cancels out factor of } h, \text{ evaluates limit)} \end{cases}$

Note: A question involving such limits (i.e., evaluating the limit definition of the derivative) is unlikely on the free-response section of the AP Exam.

$$15. (a) \quad h(x) = \frac{f(x)}{g(x)}$$

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\begin{aligned} h'(2) &= \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} \\ &= \frac{(5)(1) - (-3)(-2)}{5^2} \\ &= -\frac{1}{25} \end{aligned}$$

$$(b) \quad j(x) = f(g(x))$$

$$j'(x) = f'(g(x))g'(x)$$

$$\begin{aligned} j'(2) &= f'(g(2))g'(2) \\ &= f'(5)(-2) \\ &= (7)(-2) \\ &= -14 \end{aligned}$$

$$(c) \quad k(x) = \sqrt{f(x)}$$

$$k'(x) = \frac{1}{2\sqrt{f(x)}} \cdot f'(x)$$

$$\begin{aligned} k'(5) &= \frac{1}{2\sqrt{f(5)}} \cdot f'(5) \\ &= \frac{1}{2\sqrt{4}} \cdot 7 \\ &= \frac{7}{4} \end{aligned}$$

3 pts: $\begin{cases} 2 \text{ pts: uses the Quotient Rule to compute } h'(x) \\ 1 \text{ pt: answer} \end{cases}$

Note: Leaving the answer as $\frac{(5)(1) - (-3)(-2)}{5^2}$ is

perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question. Students are encouraged to not simplify answers on the free-response questions to avoid the risk of making arithmetic mistakes.

3 pts: $\begin{cases} 2 \text{ pts: uses the Chain Rule to compute } j'(x) \\ 1 \text{ pt: answer} \end{cases}$

Note: Leaving the answer as $(7)(-2)$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.

3 pts: $\begin{cases} 2 \text{ pts: uses the Chain Rule to compute } k'(x) \\ 1 \text{ pt: answer} \end{cases}$

Note: Leaving the answer as $\frac{1}{2\sqrt{4}} \cdot 7$ is perfectly

acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.

16. (a) (i) Because $v(t) > 0$ when $0 < t < 1$ and $4.4 < t < 5$, the particle is moving upward on the intervals $(0, 1)$ and $(4.4, 5)$.
- (ii) Because $v(t) < 0$ when $2 < t < 4.4$, the particle is moving downward on the interval $(2, 4.4)$.
- (iii) Because $v(t) = 0$ when $1 < t < 2$, the particle is at rest on the interval $(1, 2)$.

- (b) (i) When $t = 0.75$, the slope of the line of $v(t) = -2$. So, the acceleration of the particle is -2 feet per second squared.
- (ii) When $t = 4.2$, the slope of the line of $v(t) = 5$. So, the acceleration of the particle is 5 feet per second squared.

17. (a) $g(x) = f(x)\tan x + kx$
 $g'(x) = f'(x)\tan x + \sec^2 x f(x) + k$
 Because $\tan \frac{\pi}{2}$ and $\tan \frac{3\pi}{2}$ are undefined and $\sec^2 \frac{\pi}{2}$ and $\sec^2 \frac{3\pi}{2}$ are undefined, the derivative of g will fail to exist when $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

- (b) $g'(x) = f'(x)\tan x + \sec^2 x f(x) + k$
 $g'\left(\frac{\pi}{4}\right) = f'\left(\frac{\pi}{4}\right)\tan \frac{\pi}{4} + \left(\sec \frac{\pi}{4}\right)^2 f\left(\frac{\pi}{4}\right) + k$
 $6 = (-2)(1) + (\sqrt{2})^2(4) + k$
 $6 = 6 + k$
 $k = 0$

- 4 pts: $\begin{cases} 2 \text{ pts: (i) answers with explanation, where } v(t) > 0 \\ 1 \text{ pt: (ii) answer with explanation, where } v(t) < 0 \\ 1 \text{ pt: (iii) answer with explanation, where } v(t) = 0 \end{cases}$

Notes: Though open intervals are desired, open or closed intervals would generally be accepted here.

For (i) and (ii): Because the zero of $v(t)$ on the interval $[4, 5]$ is being determined visually from the given graph, any t -value in the interval $[4.3, 4.5]$ would be acceptable for this zero. (Using exactly $t = 4.4$ in the answers is not required.)

$t = 4.4$ is not included as part of the answer to part (iii) because the directions ask for open intervals where the particle is at rest. (The open interval $(4.4, 4.4)$ would be the empty set.)

- 5 pts: $\begin{cases} 2 \text{ pts: (i) answer with analysis (computes slope of this line segment)} \\ 2 \text{ pts: (ii) answer with analysis (computes slope of this line segment)} \\ 1 \text{ pt: units for both (i) and (ii)} \end{cases}$

Note: Leaving the answers in unsimplified forms, such as $\frac{2-0}{0-1}$ and $\frac{3-(-2)}{5-4}$, is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answers on a free-response question.

- 7 pts: $\begin{cases} 3 \text{ pts: uses the Product Rule to compute } g'(x) \\ 2 \text{ pts: answers} \\ 2 \text{ pts: justification (explaining } \tan x \text{ and/or } \sec x \text{ are undefined here)} \end{cases}$

- 2 pts: $\begin{cases} 1 \text{ pt: sets up correct equation} \\ 1 \text{ pt: answer (solves for } k) \end{cases}$

Note: Leaving the answer as

$$k = 6 - (-2)\tan \frac{\pi}{4} - \left(\sec \frac{\pi}{4}\right)^2(4)$$

is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.