AP® Exam Practice Questions for Appendix H

1. $\mathbf{r}(t) = \left\langle 5t^2 - 2t, 3 - \ln t \right\rangle$

$\mathbf{r}'(t) = \left\langle 10t - 2, -\frac{1}{t} \right\rangle$

$-\frac{1}{3}\mathbf{r}'(2) = -\frac{1}{3}\left\langle 10(2) - 2, -\frac{1}{2} \right\rangle = -\frac{1}{3}\left\langle 18, -\frac{1}{2} \right\rangle = \left\langle -6, \frac{1}{6} \right\rangle$

So, the answer is D.

2. $\mathbf{r}(t) = \left\langle e^{-4t} + 9, 7 - 4t^3 \right\rangle$

$\mathbf{r}'(t) = \left\langle -4e^{-4t}, -12t^2 \right\rangle$

$\mathbf{r}''(t) = \left\langle 16e^{-4t}, -24t \right\rangle$

So, the answer is C.

3 $\mathbf{v}(t) = \left\langle x'(t), y'(t) \right\rangle = \left\langle 3t^2 - \frac{1}{2}, 3(3t - 2)^2(3) \right\rangle = \left\langle 3t^2 - \frac{1}{2}, 9(3t - 2)^2 \right\rangle$

$\mathbf{a}(t) = \left\langle 6t, 18(3t - 2)(3) \right\rangle = \left\langle 6t, 54(3t - 2) \right\rangle$

$\mathbf{a}(1) = \left\langle 6(1), 54(3 \cdot 1 - 2) \right\rangle = \left\langle 6, 54 \right\rangle$

So, the answer is D.

4. The total distance traveled is

$\int_1^4 \sqrt{\left(7.5e^{-t}\right)^2 + \cos^2\left(\sqrt{t} - 1\right)}\, dt \approx 3.753$

So, the answer is B.

5. (a) $\dfrac{dx}{dt} = \ln\left[5 + (1+t)^3\right]$ $\qquad \dfrac{dy}{dt} = 4t - 3t^2$

$$\frac{d^2x}{dt^2} = \frac{1}{5 + (1+t)^3} \cdot 3(1+t)^2 \qquad \frac{d^2y}{dt^2} = 4 - 6t$$

$$= \frac{3(1+t)^2}{5 + (1+t)^3}$$

$$\mathbf{a}(t) = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle = \left\langle \frac{3(1+t)^2}{5 + (1+t)^3}, 4 - 6t \right\rangle$$

$$\mathbf{a}(2) = \left\langle \frac{3(1+2)^2}{5 + (1+2)^3}, 4 - 6(2) \right\rangle = \left\langle \frac{27}{32}, -8 \right\rangle$$

$$\text{Speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{\left(\ln\left[5 + (1+t)^3\right]\right)^2 + \left(4t - 3t^2\right)^2}$$

At $t = 2$, the speed is

$$\sqrt{\left(\ln\left[5 + (1+2)^3\right]\right)^2 + \left[4 \cdot 2 - 3(2)^2\right]^2} \approx 5.2926.$$

2 pts: {1 pt: finds acceleration vector at $t = 2$; 1 pt: finds speed at $t = 2$}

Notes: Stating $\mathbf{a}(2) = \langle x''(2), y''(2) \rangle$ is sufficient justification. The components of this vector can then be approximated on your calculator using the given $x'(t)$ and $y'(t)$ without showing further work.

Writing Speed $= \sqrt{\left(x'(2)\right)^2 + \left(y'(2)\right)^2}$ is sufficient justification. This value can then be approximated on your calculator without showing further work.

Be sure to round each answer to at least three decimal places to receive credit on the exam.

Use "≈" rather than an equal sign in presenting these approximations from your calculator. Because these are approximations, a point may be deducted if an equal sign is used.

(b) $x(0) + \int_0^3 \left(\dfrac{dx}{dt}\right) dt = 4 + \int_0^3 \ln\left[5 + (1+t)^3\right]$

$$\approx 4 + 9.0279 = 13.0279$$

2 pts: {1 pt: integral; 1 pt: answer}

Notes: Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

Be sure to round each answer to at least three decimal places to receive credit on the exam.

(c) $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{4t - 3t^2}{\ln\left[5 + (1+t)^3\right]}$

At $t = 3$, $\dfrac{dy}{dx} = \dfrac{4(3) - 3(3)^2}{\ln\left[5 + (1+3)^3\right]} = -\dfrac{15}{\ln 69} \approx -3.5427.$

So, an equation of the tangent line is

$$y - 2 = -3.5427(x - 13.0279)$$
$$y = -3.5427x + 48.1535.$$

3 pts: {1 pt: considers $\dfrac{dy/dt}{dx/dt}$; 1 pt: slope at P; 1 pt: equation of tangent line at P}

Note: Be sure to round each answer to at least three decimal places to receive credit on the exam. Use more than three decimal places when approximating the slope at $t = 3$ in the intermediate step.

(d) $dy/dt = 4t - 3t^2 = 0 \Rightarrow t = 0, 4/3$

$dx/dt = \ln\left[5 + (1+t)^3\right] = 0 \Rightarrow t \approx -2.587$

Because there is no t-value for which both dy/dt and dx/dt equal 0, the particle is never at rest.

2 pts: {1 pt: considers where $dx/dt = 0$ and $dy/dt = 0$; 1 pt: answer with reason}

6. (a) $x(3) = x(1) + \int_1^3 e^{0.5t}\,dt$

$= 4 + \int_1^3 e^{0.5t}\,dt$

≈ 9.666

3 pts: $\begin{cases} \text{1 pt: definite integral} \\ \text{1 pt: uses initial condition} \\ \text{1 pt: answer} \end{cases}$

Note: Be sure to write down the appropriate integral before numerically approximating it on your calculator.

(b) $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\sin^2 t}{e^{0.5t}}$

$\dfrac{\sin^2 t}{e^{0.5t}} = 0.25$

$t \approx 0.624$

2 pts: $\begin{cases} \text{1 pt: sets } \dfrac{dy/dt}{dx/dt} = 0.25 \\ \text{1 pt: answer} \end{cases}$

(c) $\sqrt{\left(e^{0.5t}\right)^2 + \left(\sin^2 t\right)^2} = 2$

$t \approx 1.183$

2 pts: $\begin{cases} \text{1 pt: sets expression for speed equal to 2} \\ \text{1 pt: answer} \end{cases}$

(d) $\int_0^3 \sqrt{\left(e^{0.5t}\right)^2 + \left(\sin^2 t\right)^2}\,dt \approx 7.228$

2 pts: $\begin{cases} \text{1 pt: definite integral} \\ \text{1 pt: answer} \end{cases}$

Note: Be sure to write down the appropriate integral before numerically approximating it on your calculator.

Notes: Be sure to round each answer to at least three decimal places to receive credit on the exam.

Use "≈" rather than an equal sign in presenting these approximations from your calculator. Because these are approximations, a point may be deducted if an equal sign is used.